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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  A mathematical model for the probability of penetration of the unprotected human skull is presented which relates the energy of the projectile to the properties of the skull. The model is specifically for steel spheres and cubes but can be used for other projectile shapes or densities provided that care is taken. Constants are obtained for the probabilistic model which allow calculation of the probability of penetration or, conversely, the velocity for a given probability of penetration for a given projectile/skull thickness combination.			

## PREFACE

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# LIST OF SYMBOLS

Symbol	Representing	Units
A	{ mean presented area of projectile area of top of truncated cone	cm <sup>2</sup>
a,b	constants in logistic function	
d	tensile elastic limit of skull	cm
D	$2\sqrt{A/\pi}$ (diameter of sphere of same mean presented area)	cm
m	mass of projectile	kg
P	probability of penetration of skull	
r	{ radius of top surface of truncated cone (main text) polar co-ordinate (Appendix A)	cm
R	radius of base of truncated cone	cm
s	length of side of cone	cm
S	surface area of the side of the cone	cm <sup>2</sup>
t	mean thickness of skull under area of impact	cm
T	tensile strength of skull	N cm <sup>-2</sup>
v	velocity of projectile	m s <sup>-1</sup>
v <sub>50</sub>	velocity at which there is a 50% chance of skull penetration	m s <sup>-1</sup>
X	model variable (defined in equation 4)	
X <sub>50</sub>	model variable corresponding to v <sub>50</sub> (= exp(-a/b))	
Ψ	half angle of the truncated cone	degrees
θ	polar co-ordinate (Appendix A)	degrees
φ	polar co-ordinate (Appendix A)	degrees

# A MATHEMATICAL MODEL OF THE PROBABILITY OF PERFORATION OF THE HUMAN SKULL BY A BALLISTIC PROJECTILE

## 1. INTRODUCTION

In civilian, as well as military environments, there are sometimes hazards from ballistic fragments. Because of the extreme vulnerability of the brain to penetrating injury, the difference between a relatively harmless wound and a lethal one often hinges on whether the skull is perforated when struck by one of these projectiles. In the military application, an important part of determining the vulnerability of the soldier or, conversely, the effectiveness of protective helmets lies in our ability to predict the circumstances under which the skull will withstand the impact of a fragment or bullet. In this report a mathematical model is developed which provides an estimate of the probability of perforation of the inner table of the skull and subsequent penetration of the brain by a cube or sphere. This probability is considered a function of the striking velocity and other physical properties of the projectile as well as the thickness of the bone under the impact area.

## 2. DERIVATION OF THE MODEL

The skull is composed of inner and outer surfaces of hard compact bone and an area between, called cancellous bone, which is less dense because it contains a higher percentage of living cells. The impacting projectile, when it perforates, punches out a hole in the outer table roughly its own size. The area over which the force is exerted spreads within the cancellous portion, causing a larger and more irregular hole in the inner table. This phenomenon is often called "cratering" and is used by forensic pathologists to tell exit from entrance wounds in the skull. This is possible because the larger, irregular hole is in the outer table with an exit wound.

To simplify the mathematical model, this phenomenon is idealized by the following assumptions:

1. In the region of the impact the skull is a flat homogeneous plate of bone with uniform mechanical properties.
2. When the skull is perforated by a projectile, a truncated right circular cone of bone is driven out in front of the fragment.
3. The top of the truncated cone has an area equal to the mean presented area of the projectile; i.e., the projectile orientation is assumed to be random.
4. The velocity of sound in bone is much greater than the velocity of perforation, so compressional effects may be neglected.
5. The probability of perforation depends only on the logarithm of the ratio of the stress imposed on the skull by the impacting projectile to the tensile strength of the bone.
6. The underlying probability distribution governing perforation is well approximated by the logistic distribution on the model variable, the log stress ratio of assumption 5 (e.g., if the true distribution were Gaussian this assumption would be valid). Some of the factors contributing to the variance of the probability distribution are variations in the mechanical properties of bone from skull

to skull, differences in the proportion of cortical to cancellous bone, and random deviations from some of the assumptions listed above (for instance, deviation from conical shape in the crater), each of which is unique.

A schematic diagram of the penetrating projectile is shown in figure 1. For purposes of illustration, the projectile is idealized as a right circular cylinder impacting end-on. The terms which will be used in developing the model, most of which appear in figure 1, are defined in the list of symbols. The units used are not SI but are those customarily used in wound ballistics and are retained here as a matter of convenience.

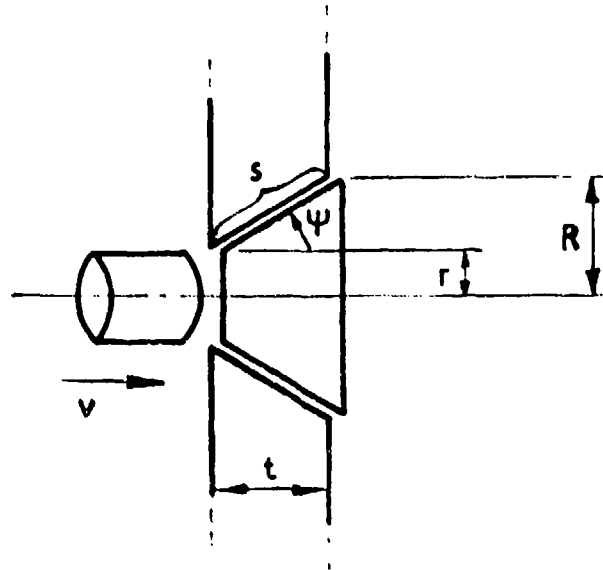


Figure 1. Idealized Schematic Diagram of Cylinder Perforating Skull

The surface area of the side of the truncated cone is given by:

$$S = \pi (r + R)s \quad (1)$$

But  $s = t \sec\psi$  and  $R = r + t \tan\psi$ , so

$$S = \pi (2r + t \tan\psi)t \sec\psi \quad (2)$$

The minimum average force exerted in pushing out the conical section of skull may be approximated by dividing the kinetic energy of the projectile by the hypothetical elastic limit of the bone, i.e., the distance that the plug could be moved before it would break loose from the surrounding material. Thus,

$$\text{force} \approx \frac{1}{2}mv^2/d$$

$$\text{tensile stress} \approx \text{force}/S$$

$$\text{stress ratio} = \text{tensile stress}/\text{tensile strength (T)}$$

$$= \frac{\frac{1}{2}mv^2/d}{T \pi (2r + t \tan) t \sec \Psi} \quad (3)$$

The quantity  $2r$  may be replaced by the variable  $D = 2\sqrt{A/\pi}$  to generalize to a projectile with a noncircular presented area. When inserted into the probability function, the multiplicative constants of equation 3 may be absorbed into logistic parameter  $a$  (see equation 5 below). With these modifications the stress ratio becomes the model variable,

$$X = \frac{\frac{1}{2}mv^2}{t^2 \sec \Psi (D/t + \tan \Psi)} = \frac{\frac{1}{2}mv^2/t^2}{\sec \Psi (D/t + \tan \Psi)} \quad (4)$$

We will use the logistic probability distribution function to estimate the probability of penetration of the skull.

$$P = \frac{1}{1 + e^{-(a + b \ln X)}} \quad (5)$$

where  $a$  and  $b$  are determined from data (see next section).

The natural logarithm ( $\ln$ ) of  $X$  is used instead of  $X$  itself to equalize the variance over the domain of definition. This requirement is most easily explained by use of an example: Suppose we are concerned with the probability that a certain projectile will perforate some material in sheets of different thicknesses where the probability of perforation is a function of velocity alone. On a thin sheet where the  $v_{50}$ , the velocity at which the probability of perforation is 0.5, is 100 m/s, we would expect a standard deviation on the order of tens of meters per second. On a much thicker sheet, where the  $v_{50}$  is 1000 m/s, we no longer expect the standard deviation to be measured in tens, but in hundreds of meters per second. In other words, we expect the standard deviation to be roughly proportional to the magnitude of the mean. In this case, the variance may be approximately equalized over the whole domain by dealing with the logarithms of the numbers rather than with the numbers themselves.

---

\* The equation is expressed in this manner as an aid to plotting  $X$  at a later stage.



### 3. FITTING THE MODEL

The data used were obtained from two previous studies on skull penetration.\*,\*\* Since the analysis on these studies showed that the response of dried skulls was significantly different from that of fresh skulls, only the latter were used in determining the parameters  $a$  and  $b$  for the logistic function of equation 5. The physical properties of the five projectiles used are listed in the table.

Table. Projectiles Studied (All Made of Steel)

Projectile	Mass	Mean dimension*	Mean presented area**
	gm	cm	cm <sup>2</sup>
0.85 grain sphere	0.055	0.238	0.0445
2.1 grain cube	0.135	0.265	0.1050
4.2 grain cube	0.274	0.333	0.1660
16 grain cube	1.029	0.514	0.3966
225 grain cube	14.694	1.236	2.2933

\* Diameter for sphere; edge for cubes.

\*\* Surface area/4; see appendix A.

The ability of the model to properly scale the probability of perforation of skulls of different thickness by projectiles of different mass was tested against the  $v_{50}$  of the above five projectiles. The  $v_{50}$  was obtained by averaging the five highest nonperforating velocities and the five lowest perforating velocities for each skull thickness for which there were sufficient data.† These data are shown plotted in terms of the model in figure 2. Note that the abscissa is the logarithm of the numerator of the model variable of equation 4, while the ordinate is the logarithm of the denominator. A straight line of slope 1 represents a constant ratio of the numerator and denominator (see appendix B). When drawn through the data in figure 2, this line allows a visual assessment of the goodness of fit of the data from low energy to high energy. The energies are  $\frac{1}{2}mv_{50}^2$  for all  $v_{50}$  points. Since  $\Psi$  is a variable, it may be adjusted to cause the sphere and cube data to be on a common line. A  $\Psi$  of  $23^\circ$  for the sphere and  $30^\circ$  for the cubes not only superimposed the two groups but tilted the individual data sets to form the best straight line of slope 1. Since the co-linearity of the data with the reference line is relatively insensitive to  $\Psi$ , the difference ( $7^\circ$  between spheres and cubes) is more significant than the magnitude of  $\Psi$ .

\* Miller, J. E., Ashman, W. P., and Jameson, J. W. Edgewood Arsenal Technical Report EATR 4373. Ballistic Limits of Skulls Against Steel Cubes. April 1970.

\*\* Mickiewicz, A. P. Chemical Systems Laboratory. Private communication.

† To obtain the maximum number of  $v_{50}$  points, a few were averages of 4 and 4 instead of 5 and 5.

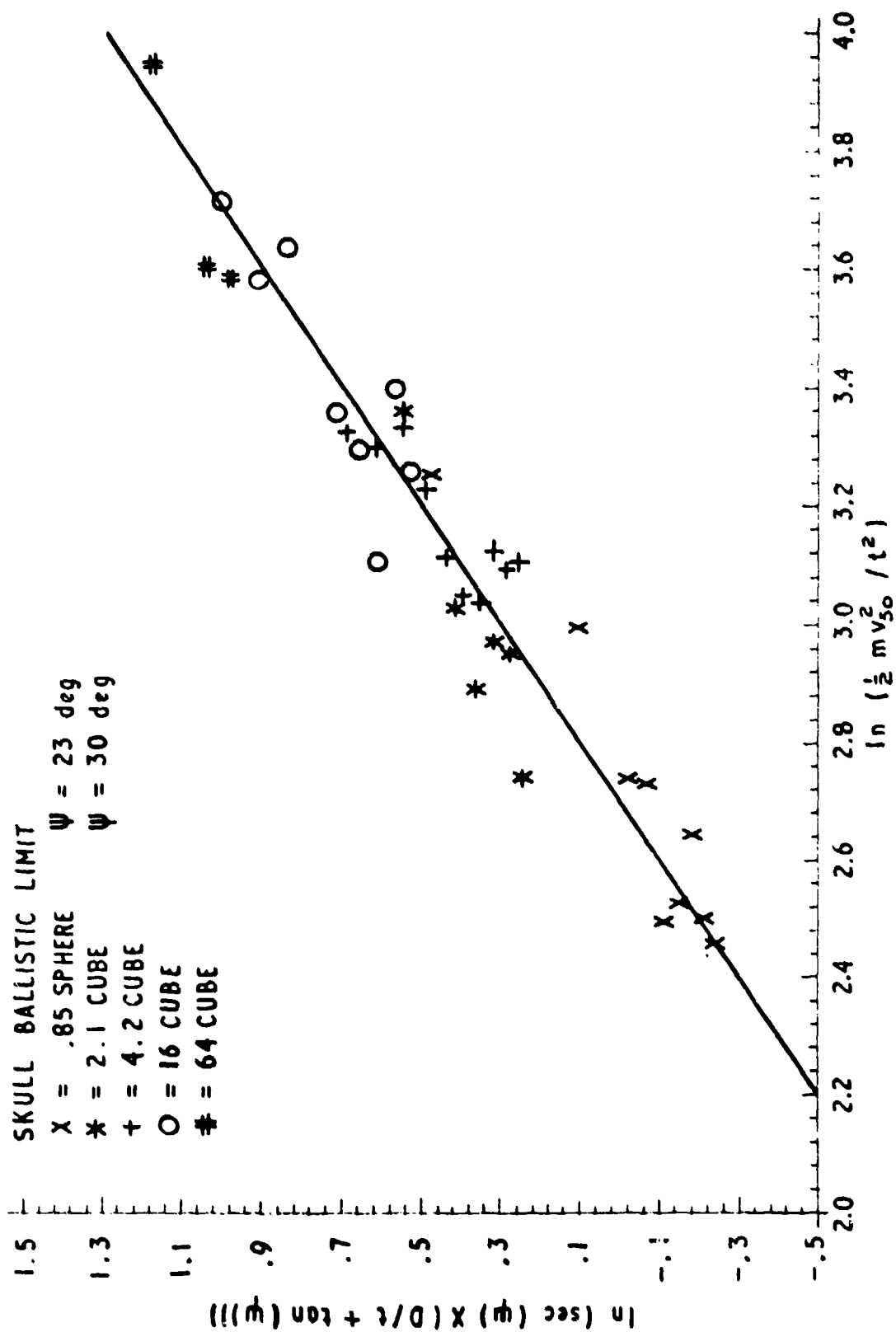


Figure 2. Plot of the Two Components of the Model Variable X With  $V_{50}$  Line

After the determination of  $\Psi$ , all the variables in equation 4 are known. It is then possible to fit the dichotomous data (perforation or nonperforation) to the logistic function by the approximate least squares method of Walker and Duncan.\* In this technique all data are used to approximate the  $v_{50}$  for each skull thickness/projectile combination. For each shot the mass, striking velocity,  $\Psi$ , skull thickness, and mean presented area are used to calculate  $X$  from equation 4. A second number is associated with this value: 1 for perforation, 0 for nonperforation. The Walker-Duncan method then iteratively converges on values of  $a$  and  $b$  which approximately minimize the sum of squared differences between the predicted probability of penetration (equation 5) and the assigned value (0 to 1) for all shots by all projectiles. The resulting values of  $a$  and  $b$  are:

$$a = -13.035, b = 4.793$$

Lines of slope 1 in figure 2 represent constant values of  $X$  and, therefore, discrete levels of probability of perforation. To find the line of 50% probability we insert  $P = 0.5$  in equation 5 and solve.

$$P = \frac{1}{1 + e^{-(a + b \ln X_{50})}} = 0.5$$

or

$$e^{-(a + b \ln X_{50})} = 1$$

This implies

$$a + b \ln X_{50} = 0$$

$$\ln X_{50} = \ln \frac{\frac{1}{2} m v_{50}^2}{t^2 \sec^2 \Psi (D/t + \tan \Psi)} = -a/b \quad (6)$$

where  $X_{50}$  is from equation 4 with  $v = v_{50}$ .

Therefore,

$$\ln[t^2 \sec^2 \Psi (D/t + \tan \Psi)] = a/b + \ln[\frac{1}{2} m v_{50}^2] \quad (7)$$

It is this line which is drawn in figure 2. The good agreement between the line, derived from all data, and the plotted points, derived from a few selected data points, lend some credence to the validity of using those approximate  $v_{50}$  points in determining the value of  $\Psi$ . An example of the use of the model is given in appendix C.

\* Walker, S. H., and Duncan, D. B. Estimation of the Probability of an Event as a Function of Several Independent Variables. *Biometrika* 54, 167-179 (1967).

#### 4. SUMMARY AND CONCLUSIONS

The model derived above is based on the assumption that a plug of homogeneous material in the shape of a truncated cone is driven out of the skull by the perforating projectile. The area of the small end of the truncated cone is assumed equal to the mean presented area of the projectile. Under these assumptions, both spheres and cubes may be accommodated in the same model by letting the half angle of the cone take on different values for the two shapes. Experience with previous models of penetration has shown that chunky, irregular fragments usually behave in a manner more similar to cubes than spheres. Therefore, it is suggested that the cube half angle,  $\Psi = 30^\circ$ , be used for fragments as well.

The data used in fitting this model were obtained from impacts on bare bone. In predicting the probability of penetrating the skull of live humans, this model neglects the protection offered by the scalp and (sometimes) hair. Thus the model is conservative in the sense of predicting somewhat higher probabilities of penetration that would actually be observed in live human skulls of the same thickness.

Caution must be exercised when applying these criteria to projectiles larger than those in the data base or of density much different from that of steel. There are no data available to test the model outside the range of masses and presented areas of the projectiles of the table. Large projectiles at lower velocities are particularly risky because of significant curvature of the skull over large areas and different mechanisms of skull fracture which occur under these conditions. It should also be noted that assumption 3 requires random orientation of the impacting projectile. This will be valid for projectiles whose mean dimension is not much larger than the thickness of the skull, since in perforating the skull the projectile will ultimately present its mean area, e.g., by rotation during penetration. For projectiles whose mean dimension is greater than the skull thickness (for example, paper weights), the projectile is unlikely to perforate presenting its mean area, and the effect of orientation of the projectile will be important.

#### 5. RECOMMENDATIONS FOR FURTHER WORK

When examining the skullcaps perforated by cubes, one does not see flat-sided holes with neat square corners, but rounded holes. This indicates that the assumption of equality between the mean presented area of the projectile and the area of the hole in the outer table should instead be an assumption of proportionality. Differences between spheres and cubes could then be accounted for by different constants of proportionality rather than different half angles  $\Psi$ . Irregular fragments would have a proportionality constant somewhere between those of the cube and sphere. Unfortunately, there are no data available for irregular fragments nor even sufficient data for the spheres and cubes to allow an accurate determination of the proportionality constants.

If additional data are acquired, particularly fragment data, it is suggested that the following model be tried:

$$X = \frac{\frac{1}{2} mv^2}{t(kD + t \tan \Psi)}$$

where  $k$  depends on the shape of the projectile (sphere, cube, chunky fragment, etc.) and  $\Psi$  is a constant, independent of shape.

## APPENDIX A

### MEAN PRESENTED AREA

Let us first develop the "simple case" of the mean presented area of a sphere. We want to express this as a fraction of its total surface area. To obtain the surface area we will integrate in polar co-ordinates as in figure A-1. Notice that the width of the element of area  $dA$  is  $r \sin \theta \, d\phi$ . The sine function is necessary because, like the outer surface of an orange segment, it must narrow to a point at the top ( $\theta = 0$ ). Even if the wedge is infinitesimally thin, it still must be shaped like a wedge. Another way of looking at this effect is to imagine  $\theta$  held constant; then when we rotate around the vertical ( $z$ ) axis through an angle  $d\phi$ , the distance moved on the surface is  $r \sin \theta \, d\phi$ . The surface area of the sphere is

$$\begin{aligned} A_S &= \int_0^{2\pi} \int_0^\pi r^2 \sin \theta \, d\theta \, d\phi = r^2 \int_0^{2\pi} (-\cos \theta) \Big|_0^\pi d\phi \quad (A-1) \\ &= 2r^2 \int_0^\pi d\phi = 4\pi r^2 \end{aligned}$$

Referring again to figure A-1, we now wish to obtain the mean area projected on the plane P1 at the right of the figure, parallel to the  $x$ - $z$  plane. The width and height of  $dA$  as projected on this plane are  $r \sin \theta \sin \phi \, d\phi$  and  $r \sin \theta \, d\theta$ . Thus the projected area is

$$A_P = \int_0^\pi \int_0^\pi r^2 \sin \phi \sin^2 \theta \, d\theta \, d\phi \quad (A-2)$$

where the limits of integration on  $\phi$  are 0 and  $\pi$  because the region from  $\pi$  to  $2\pi$  is on the far side of the sphere from the plane P1 and does not contribute to the presented area required. Thus,

$$\begin{aligned} A_P &= r^2 \int_0^\pi \sin \phi \left[ \frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right]_0^\pi d\phi \\ &= \frac{\pi r^2}{2} \int_0^\pi \sin \phi \, d\phi = \pi r^2 \quad (A-3) \end{aligned}$$

The mean presented area  $A_P$  (as expected) is the same as the cross-sectional area and is a quarter of the surface area. The angles  $\theta$  and  $\phi$  may be interchanged without affecting the results.

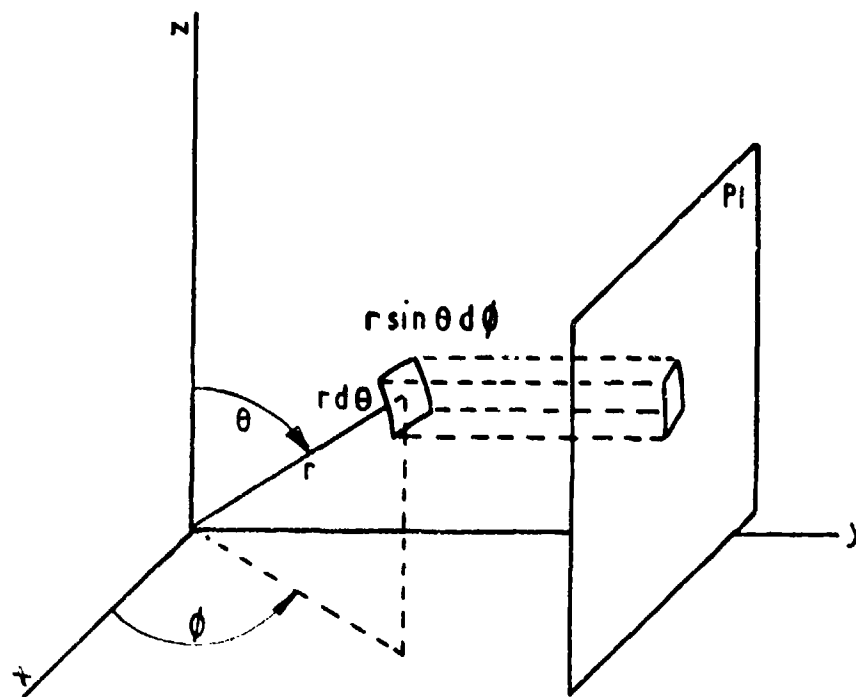


Figure A-1. Element of Area of a Sphere and Its Projection on a Plane

We now turn to the problem of the mean presented area of a polyhedron with an arbitrary number  $q$  of plane elements as its surface. Let the area of a typical element be  $E$ . Then the projected area of the element  $E$ , under all possible orientations of the polyhedron with respect to the plane of projection ( $x$ - $z$ ), is equivalent to the projected area of  $E$  rotated about its center of mass as in figure A-2. The co-ordinate system is centered at the center of mass in such a way that the normal  $\underline{n}$  to the area  $E$  makes an angle  $\theta$  with the  $z$ -axis. The projection of the normal onto the  $x$ - $y$  plane makes an angle  $\phi$  with the  $x$ -axis. The integral of the area projected onto  $P1$  is

$$A_I = E \int_0^\pi \int_0^\pi \sin\theta \sin\phi \, d\phi \sin\theta \, d\theta \quad (A-4)$$

$$= E \int_0^\pi \int_0^\pi \sin\phi \sin^2\theta \, d\theta \, d\phi = \pi E$$

where the first  $\sin\theta$  is, as above, necessary to provide equal weight to all possible orientations. Because  $E$  is a finite area (not an infinitesimal), the area  $A_I$  is an integral projected area - not a mean. To obtain the mean, we must divide  $A_I$  by the total solid angle subtended by all possible orientations of the element  $E$  (i.e., the solid angle swept out by  $\underline{n}$ ), namely  $4\pi$ . Thus the mean projected area  $A_p = A_I/4\pi = E/4$ .

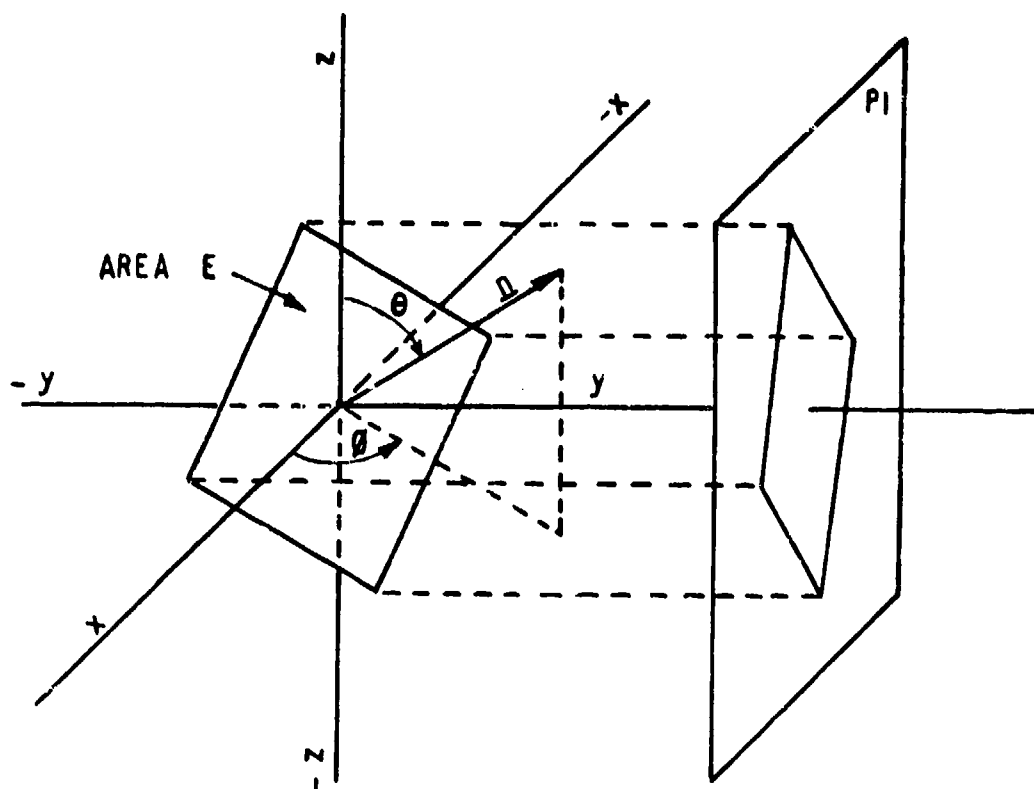


Figure A-2. Element of Area of a Polyhedron  
and Its Projection on a Plane

If there are  $q$  elemental areas in the polyhedron each having a mean projected area equal to a quarter of its surface area, the total mean projected area is a quarter of the total surface area, i.e.,

$$A_p = \sum_{i=1}^q A_i/4 = \frac{1}{4} \sum_{i=1}^q A_i = \frac{1}{4} A_s \quad (A-5)$$

This formula is valid for convex surfaces only. If there were a concave portion on the surface, it would be shielded by another part of the surface on part of the interval  $0-\pi$  of one or both angles of equation A-4. Any convex shape can be approximated to any specified accuracy by making  $q$  sufficiently large and the  $A_i$  sufficiently small. Thus the mean presented area of a convex solid is  $\frac{1}{4}$  of its total surface area.

## APPENDIX B

### COMMENTS ON THE SLOPE OF A LOG - LOG PLOT

Consider the graph in the figure, where the two variables  $N$  and  $D$  are plotted on logarithmic axes. Suppose the data indicate a straight line relationship on this graph.

Then,

$$\ln D = p \ln N + q$$

$$= \ln N^p + q$$

$$\therefore \ln N^p - \ln D = -q$$

$$\therefore \ln \left( \frac{N^p}{D} \right) = -q$$

$\therefore$  after exponentiating,

$$\frac{N^p}{D} = e^{-q} = \frac{1}{q'}$$

If  $p = 1$ , there is a constant ratio between  $N$  and  $D$ .

If  $p = 2$ , there is a constant ratio between  $N^2$  and  $D$ .

If  $N = \frac{1}{2}mv^2/t^2$  and  $D = \sec\Psi(D/t + \tan\Psi)$  the  $N/D = X$  (see equation 4). A line of slope 1 in the plot of  $\ln N$  v  $\ln D$  then represents a constant ratio between  $\frac{1}{2}mv^2/t^2$  and  $\sec\Psi(D/t + \tan\Psi)$ .

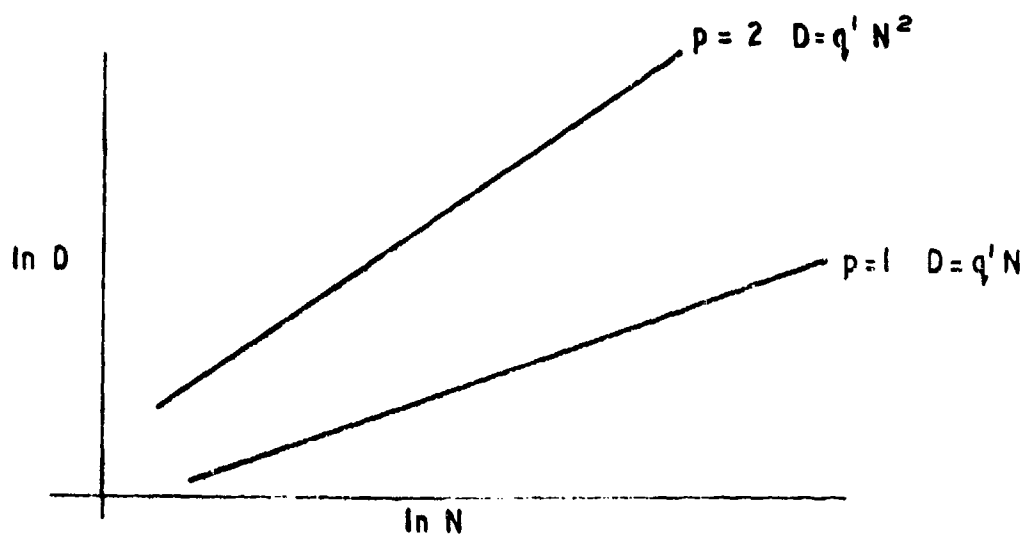


Figure. General Linear Relationship Between  $\ln N$  and  $\ln D$



## APPENDIX C

### EXAMPLE OF THE USE OF THE MATHEMATICAL MODEL

Let us assume that we wish to know the velocity corresponding to a 0.5 probability of penetration for a 0.5-gram cube of mean presented area  $0.245 \text{ cm}^2$  impacting a skull of 0.5-cm thickness. From equation 6 we obtain

$$v_{50} = [2t^2 \sec\Psi(D/t + \tan\Psi)\exp(-a/b)/m]^{1/2}$$

where

$$D = 2\sqrt{A/\pi} = 2\sqrt{0.245/\pi} = 0.5585 \text{ cm}$$

$$\Psi = 30^\circ$$

$$t = 0.5 \text{ cm}$$

$$m = 0.0005 \text{ kg}$$

$$a = -13.035$$

$$b = 4.793$$

Substitution above gives  $v_{50} = 172 \text{ m/s}$ . Suppose we wish to know the probability of penetrating the skull if the cube of our example impacts at 200 m/s. From equation 4 we obtain

$$X = \frac{\frac{1}{2}mv^2}{t^2 \sec\Psi(D/t + \tan\Psi)} = 20.44$$

Then

$$P = \frac{1}{1 + \exp(-a - b\ln X)} = 0.807$$

The following table can be constructed using similar calculations, assuming the same fragment and skull thickness:

Velocity m/s	Probability of penetration
150	0.209
175	0.537
200	0.807
225	0.973

The value of probability of penetration above does not indicate the proportion of the skull thickness penetrated but rather the proportion of hits which would be expected to punch out a plug of bone

from the skull given a large number of hits. Thus, in the example above, at  $v = 200$  m/s, 80.7% of all shots fired would be expected to penetrate for that fragment/skull thickness combination, while 19.3% would not. No attempt is made to predict the outcome of individual shots.

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US Public Health Service  
Center for Disease Control  
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Building 4, Room 232  
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